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Preface

Professor Aparna Ganguli and Richard Henry have generously agreed to publish the bibliography of sources they discovered as they conducted research into the uses of writing in mathematics classes as a part of a grant sponsored by the Center for Interdisciplinary Studies of Writing at the University of Minnesota. These sources should prove useful to others interested in using “writing-to-learn” approaches in similar classrooms.

The Center for Interdisciplinary Studies of Writing annually funds research projects by University of Minnesota faculty to study any of the following topics:

- characteristics of writing across the University’s curriculum;
- status reports on students’ writing ability at the University;
- the connections between writing and learning in all fields;
- the characteristics of writing beyond the academy;
- the effects of ethnicity, race, class, and gender on writing; and
- curricular reform through writing.

We make informal reports on the projects, such as this bibliography compiled by Ganguli and Henry, available in the form of technical reports. More elaborate reports and extended discussions of Center grant recipients’ work are available through our monograph series.

One of the Center’s goals is to disseminate the results of these research projects as broadly as possible within the University community and on a national level. We encourage discussion of Aparna Ganguli and Richard Henry’s annotations and interpretations of the literature currently available on using writing in mathematics classes, and we invite you to
contact the Center for Interdisciplinary Studies of Writing for information about other publications or Center activities.

Lillian Bridwell-Bowles, Series Editor
Mark Olson, Editor
January 1994
Introduction

We wanted to examine what progress had been made in integrating writing into mathematics instruction in the years since Janet Emig\(^1\) published her influential article advocating writing as a mode of learning and William Geeslin\(^2\) investigated writing as a teaching technique tailored to mathematics instruction in 1977. The literature search represented in this bibliography covers material published from 1977 through 1990 that specifically addresses writing to learn in mathematics. It is one component of the research project, Integration of Writing into College Algebra, funded by the Center for Interdisciplinary Studies of Writing at the University of Minnesota.

Although the authors of the forty-seven articles annotated here generally believe that integrating writing into the mathematics classroom helps students to a greater understanding of mathematics, very few formal research studies have been conducted to substantiate their beliefs. For the most part, the authors offer anecdotal accounts of increased student comprehension, greater communication between students and teachers, improved teacher insight into student learning, and better attitudes of both teachers and students toward mathematics.

The objective of this bibliography is to share information about the progress made by mathematics teachers who are trying to incorporate writing-to-learn-mathematics strategies in their classrooms. We also hope that this bibliography will help researchers interested in the connections between writing and learning mathematics.

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1 See Janet Emig's 1977 article, "Writing as a Mode of Learning," in *College Composition and Communication*, 28, 122-128.

2 William Geeslin's article, "Using Writing about Mathematics as a Teaching Technique," is annotated on page 6.
Bibliography


The authors believe that writing should be integrated into every mathematics class. They argue that writing in the process of learning subject matter helps students see new relationships, invent new ideas, and communicate better. They also describe a variety of writing activities, including journal writing, story problems, written algorithms, and microthemes. These activities are analyzed for their usefulness in developing students’ cognitive skills. Other written assignments ask students to contextualize subject matter by including essay questions as homework, writing test items, interviewing a mathematician, responding to hypothetical situations, and paraphrasing problems for younger students. Abel and Abel also discuss what constitutes a “good” writing assignment and the role evaluation plays in the classroom. They provide a sample assignment with an evaluation rubric based upon accuracy of details, comprehensiveness, and organization/clarity.


Azzolino discusses writing as a teaching tool in mathematics classes and offers several short in-class writing assignments such as non-thought warm-ups, lead sentences, completion, rewording, word banks, and debriefing. The author claims that these assignments are useful diagnostic tools as well as an aid to student comprehension of mathematics.

This paper focuses on correspondences between mathematical problem solving and expository writing. The authors argue that expository writing can improve Student understanding of a subject. They conducted a pilot study involving two ninth-grade general mathematics classes. The experimental group was taught using traditional math techniques as well as structured expository writing for learning problem solving skills. The control group was taught using traditional math techniques without any writing. After four weeks, the results indicated a significant difference at the .01 level between the groups, favoring the experimental group. Samples of assignments given to both groups are provided.


This article introduces a vocabulary for writing to learn in the content areas and provides definitions for such phrases as “think books,” “free writing,” and “clustering.” Boone has assembled ideas and suggestions by other educators in different levels of teaching. There is an example of evaluating content area writing using plus signs, check marks, and minus signs to indicate the quality of understanding the material as it is expressed through writing.


Borasi and Rose address the educational value of requiring a written journal in a mathematics class. The authors warn that one has to be careful about what kind of writing activities constitute writing to learn in mathematics. The article offers an in-depth discussion of potential benefits of journal writing. Their experiences suggest that journal writing helps both the teacher and the student in the teaching and learning process.
This paper reports the outcome of a college-level writing seminar in mathematics taught by the author. Each week, students wrote two two-page entries that became a mode of communication between the teacher and the students. Some of the topics were Pascal’s triangle, the Pythagorean theorem, the Fibonacci sequence, and the golden ratio. As they read textbooks and wrote their own views, students realized that there were disagreements even in mathematics.

Through the students’ writing, the author followed the development of their understanding, tried to understand the logic of their errors, and explored appropriate instructive mediation to help them. Students wrote to reflect on their ideas and feelings about mathematics. The teacher used these reflections to help the students gain confidence and power in relationship to mathematics. Much of the discussion centers around the journal entries of one math-avoidant student enrolled in the seminar.

Arguing that “getting the right answer” is not the only objective of learning mathematics, Burns suggests that incorporating writing into mathematics classrooms will help students develop their abilities to think about and understand content. Student writing also provides teachers with a better understanding of students’ cognitive processes. Using middle school students as examples, the author discusses classroom strategies such as focusing on problems requiring written responses, not just numerical answers; using word problems that get students
reasoning as well as finding answers; providing opportunities for students to present the thinking processes they used in solving a problem; and using students’ writing to assess their understanding of mathematical concepts.


Burton discusses free writing, journals, in-class writing, and term papers and describes their value with respect to the kinds of learning done in college- and graduate-level mathematics education courses. Journals, for example, go beyond ordinary class notes in that they allow students to explore personal reactions to material and teaching methods, thereby serving as a more thorough course record. In-class writing can be used as a means to clarify class notes or to explore metaphorical thinking. The author suggests peer reviews for term papers and provides a sample workshop checklist for reviewers and a five-point rating scale that explains the differences between strong and weak papers with respect to central ideas, organization, development, style, and technical control.


An invaluable collection of twenty-three essays ranging from theoretical speculation on the usefulness of writing to learn in the content areas and the relationships between mathematical and ordinary language, to classroom applications, both anecdotal and experimental.

The authors argue that writing about a topic involves many of the thought processes that should be fostered in a mathematics classroom. They have practiced various uses of writing in a mathematics classroom and classified them into five categories: direct use of language (copying and recording information), linguistic translation (from mathematical to ordinary language), summarizing (and/or paraphrasing), applied use of language (writing story problems), and creative use of language (writing that does not specifically address problem solving but which is related to mathematics). Davison and Pearce provide sample activities in each of these categories and the implications for student learning.


The authors believe “most aspects of students’ mathematical development, from concept acquisition through problem solving abilities can be facilitated through integration of writing activities.” Since textbooks play a major role in developing teaching strategies the authors considered five textbooks, commonly used at the seventh and eighth grade levels. They examined the quality and quantity of writing activities used in the textbooks and formulated a three-category classification system: response to a question, problem formulation, and report on a project. The authors found that writing was both undervalued and underrepresented in the textbooks with little or no attention paid to providing students and teachers with a writing process that includes prewriting and revision strategies.


This paper documents an experiment incorporating writing into a fifth grade mathematics course. Students were asked to describe how to draw angles and multiply; to define ray lines, and
planes; and to “troubleshoot their own homework to explain their errors. The writing component allowed the author to identify student misconceptions. The experimental class showed significant gains on test scores over a comparable section. Students who benefited most were those with lowest pre-test scores.


This paper addresses the importance of students writing word problems. The argument is that unless students are capable of writing problems on their own, they will not be able to grasp the underlying mathematical concepts. The authors discuss the writing process (prewriting, writing, rewriting, revising, and publication) and the roles teachers play at each stage.


This article discusses the outcome of an experimental research project involving fifty college students enrolled in a developmental mathematics class. The experimental group was given short in-class writing assignments. The same instructor taught the control group but without writing activities beyond ordinary note taking. The experiment ran three days a week for four weeks. Results indicated that the experimental group performed better than the control group on the final examination.


This article discusses the results of an investigation into the learning of mathematical structure and its connection to achievement. The experiment included writing tasks for students
in grades four through twelve. The results indicated that most students learned elementary probability and the mathematical structure involved, but they experienced considerable difficulty on the various writing tasks. The results also indicated that achievement tests did not always detect many important mathematical misconceptions. The author believes that practice in writing mathematical concepts improves student learning. Samples of student writing in various grade levels are included.


Goldberg discusses the importance of students writing in mathematics courses. The article also describes and evaluates “five-minute writing” assignments and essay questions on tests, as well as the importance of the writing process and peer revision.


Assuming “thought and expression of thought are so closely interrelated that to require the latter will engender the former,” the authors instituted an experimental calculus course at Duke University that integrated writing assignments. The content of the course was the standard first-year calculus syllabus. In addition to the regular class time, the students met one extra hour each week for a lab assignment. Every week, the students wrote lab reports that demonstrated their understanding of the content.

The authors discuss in considerable detail common problems with student writing in mathematics. They also discuss using writing strategies based on readers’ expectations of substance and structure. The authors paid careful attention to the revision process by requiring
rough drafts to be submitted for feedback before final drafts were submitted for a grade. Peer evaluation was also encouraged. Toward the end of the term students revised without the instructor’s responses. The authors believe the “initial experiment has proved an exciting success.”


This article discusses a teaching experiment that explored the processes on students’ error patterns. Students were given problems that involved performing operations on algebraic fractions and solving algebraic fractional equations. Of six sections of developmental elementary algebra classes, three received writing assignments asking students to identify similarities and differences between mathematical processes. The other three did not get any writing assignments, but did receive extra problems to solve. The error patterns of all sections were analyzed.

The results did not show any difference in the distributive property error occurring between the sections that wrote about the mathematical processes and those sections that did not. However, sign error was more frequent in the sections that did not write. Overall, students in the experimental group enjoyed the follow up discussion on the similarities and the differences of the problems assigned in the quiz. Goldberg notes that professors occasionally saw problem areas as a result of reading student writing. He also provides several suggestions for further studies.

This paper describes an experimental study involving an elementary algebra course. Eighty-three students were placed in experimental and control groups. Experimental subjects were given fifteen assignments requiring written explanations of the mathematical concepts inherent in the problems they had to solve. Control subjects just solved the problems. The teachers were not informed which students were assigned to which groups and an independent grader evaluated the assignments. No significant differences were found between the groups with either pre- or post-performance measures.


Johnsen argues that writing is an analytical tool and can be used for learning and teaching. Meaningful writing helps students to understand mathematical concepts better, therefore making them more proficient at problem solving. The author presents sample-writing assignments including writing story problems and book reports and suggests that teachers include an essay question on tests.


In this article, the author analyzes student-writing samples for the underlying problems they reveal. Students able to do computations are not as skilled at defining mathematical concepts and writing proofs. Keith offers a variety of writing activities classified under the headings of summaries, visual image translation, synopsizing tactics for solving a problem, giving definitions, communicating thoughts to a particular audience, inventing a problem,
generalizing a concept, and group projects. She emphasizes the need for giving feedback on student writing and the importance of incorporating group discussion, peer evaluation, and revision into the classroom writing process.


This paper discusses the results of implementing writing as a learning tool in a pre-calculus course and emphasizes the importance of writing in developing algorithmic thinking, which, as the authors note, is the “backbone of mathematics.” Admitting that computational skills are usually the first step in learning mathematics, the authors argue such skills do not necessarily indicate a deeper understanding of the subject. Writing helps in the development of this deeper understanding by improving class participation, by creating “opportunities for conceptual growth” for all students, by stimulating discussion, and by alleviating anxieties created by the feeling that math is a “special” language removed from the ordinary world. An appendix provides sample projects in areas such as expressive writing, summaries, journals, visual image translation, synopsize/organize, definition, rewrite, writing algorithms, audience awareness, and problem invention.


This paper describes how writing letters, keeping logs, creating word problems, and writing approaches to solutions were integrated into the teaching and learning of mathematics in an elementary mathematics classroom. After using this researching strategy, the author believes writing helps relieve math anxiety, clarify thinking, and generate ideas and questions.

After defining cognitive learning and its relation to problem solving, the author argues that writing is a significant method for solving problems. The author notes that although writing is an integral part of the thought process, the Writing Across the Curriculum movement is not widely received in the mathematics community. He recommends that activities should be designed to ensure that writing becomes a normal part of the daily routine in the mathematics classroom. Kenyan offers examples of long-term writing projects requiring research and short-term writing projects. Short-term projects include asking students to compare two mathematical concepts, to explain why or how a procedure might be done, to write word problems, or to outline a chapter from the textbook. The author discusses the short-term projects at some length and concludes that writing makes students be more active in their thinking processes and therefore makes them better problem solvers.


After noting Britton’s identification and definitions of three functions of writing (transactional, expressive, and poetic), King offers a taxonomy of transactional and expressive writing assignments with brief examples. Transactional assignments are divided into summaries, questions, explanations, definitions, reports, and word problems. Expressive assignments include journal writing, free writing, letter writing, and daydreaming on paper. King suggests expressive writing be included in mathematics courses in addition to transactional writing.

This research report investigates the effect of journal writing on the thinking skills of high school geometry students. The author believes that writing forced the students to synthesize information and helped them to recognize their individual learning styles. Journal writing in the teaching and learning of geometry served as a diagnostic tool for the instructor and enhanced students’ metacognitive abilities. The results of the project suggest this teaching strategy would be effective in all disciplines. The report gives a broad database of student responses and instructor notes.


Requiring students to write term papers is one way of making students become aware of the interdisciplinary nature of mathematics. Students were encouraged to use creative ways of “mathematizing” ordinary situations. The examples include examinations of the New York City subway system in terms of efficiency or profit, investigations into fast-food chains, and analyses of Federal Reserve transactions. The assignment can be structured to meet the needs of the traditional, remedial, or continuing-education students. Most of the students considered the term paper a valuable component of the course and said it should be continued.


Mett discusses integrating three kinds of writing assignments into calculus instruction: journal writing, in-class writing, and assigned projects. Assigned projects are recommended for honors or advanced students. Writing in class was not graded but was used in-group discussions.

Based on the author’s experiences with teaching college calculus classes, this paper analyzes samples of student writing and asserts that journal writing helps students develop associations between various uses of terminology, create personal definitions, and experience meaningful connections between mathematical concepts.


This paper describes and analyzes a collaborative research project which investigated the effect of regular writing in algebra classes on students’ attitudes and skills in algebra and the effect of regular reading of student writing on teacher awareness of student difficulties in and attitudes toward algebra. Contextual, instructional, reflective and miscellaneous writing prompts were developed. They were used in at least four out of every five instructional days by requiring students to write for a minimum of five minutes. Teachers spent some time each week reviewing the students’ writing. The results indicated that the students’ writing improved in quantity and quality over time. Also improved were the attitudes of the students and the teachers toward each other and about the teaching and learning of algebra. One of the most important outcomes was that teachers gained insight into their teaching through their analyses of their students’ writing.


The authors’ experiences indicate that in-class journal writing is the most effective method of using writing to help students learn mathematics. Through journal writing, the authors
attempted to develop students’ abilities to synthesize, interpret, translate, analyze, and evaluate mathematical experiences and to internalize mathematical concepts by using these intellectual skills. Samples of short in-class student writings of analogies of real life experiences to various mathematical rules are discussed. Although no strong relationships were found between writing, test scores, and student attitudes, the students felt the inclusion of journals in mathematics classes were valuable to their thinking about solving problems associated with mathematical concepts.


This paper provides samples of students writing about qualitative graphing. The author found that asking students to integrate writing into their graphing enabled their self-monitoring and metacognitive abilities. Examples of writing assignments of graphical representation requiring in-depth description of thought processes are included.


This is a collection of activities suitable for students in elementary school. The activities are meant to develop students’ understanding of the language of mathematics and mathematical concepts through reading and writing. Activities include giving instructions to robots, comparing distances traveled reading a map and writing a story with illustrations about a robot, using newspaper ads to analyze and add prices, developing menus, reading rate charts, solving problems and computing costs.

The author reports the results of an experiment linking a remedial mathematics course with a freshman composition section. Teachers collaborated in designing activities to explore the organizational patterns both in mathematics and in writing. Several sections of the remedial mathematics course served as a control group. Pre- and post-test scores showed a slight advantage to the experimental section. This advantage could be considered significant since the retention rate of weaker students was much greater for the experimental section than the control group.


The authors investigated the quantity, kinds, and uses of writing in eight junior high mathematics classrooms. The findings suggest writing is not practiced frequently in junior high mathematics classes and the kind of writing is limited to direct note taking and copying tasks rather than original student writing for instructional purposes. In general, attention to the writing process (prewriting and revision) or to the peer review was absent from the classes.


The author describes his experience in devising an elementary-level number theory course with a major writing component. He discusses guidelines for written homework, the purpose of written homework, and student responsibility. The article offers examples of common errors in homework, how English is used for mathematical communication, and how to strike a balance between words and symbols in writing. According to the author, one quarter of the class
members became reasonable writers after one semester. The author concludes that all math students should be required to write in their courses.


This study describes a writing activity in which elementary and junior high school students pick up a “problem of the week” on Monday, solve the problem during the week, and write a paragraph describing the method they used to solve the problem to hand in on Friday. Students are required to confer with a partner to revise and edit their writing. “Thinking journals,” another form of writing consisting of problem solving and problem finding sections, are also used. These writing activities help students set goals and identify strategies for solving problems.


Recognizing that many students rely on verbalization for understanding, the author argues that mathematics teachers must consider integrating writing and discussion into the learning process. This article provides samples of student-written learning logs and an explanation of how they can be used as teaching tools. Three writing assignments, “chatter,” “explanation of how,” and “explanation of why,” are discussed. Despite the lack of success students had with the third kind of assignment, the author found it did help students understand the reasons for certain mathematical concepts.

The author discusses the importance of writing in teaching mathematics and argues that not using writing in teaching mathematics means less communication with students, more emphasis on rote memorization of algorithm, and less understanding of mathematical concepts. The article discusses the three essential elements of the writing process—prewriting, writing and rewriting.


The authors detail their own shift from using writing as a display of student knowledge for evaluative purposes to writing as a learning strategy. They discuss several student writing activities: creating stories, writing mathematical essays, keeping diaries, and writing dialogues. The authors conclude that writing helps students understand, organize, and become more active in the learning process; it can be used to shift the emphasis from the goal of a final, correct answer to the actual thinking and learning that occurs in the classroom.


This book has thirty-one essays written by college-level educators; it is an important source in writing-to-learn literature in mathematics. Chapters are divided into six categories: “Introduction”; “Getting started”; “On grading or ‘to English or not to English’”; “What students say?”; “On keeping journals”; and “Course specific, but with broadly applicable ideas.”. The writers discuss Writing Across the Curriculum projects and the value of keeping journals and other forms of expressive writing. They offer specific advice on getting started with writing programs and on routine matters such as grading, correcting grammar, and the importance of
rewriting. Several essays describe students’ reactions to writing in mathematics classes and how to involve students in reading and grading others’ work.


This paper reports how writing was used in a remedial algebra classroom to enhance student understanding. Regular homework assignments were expanded into a journal of algebraic understanding that was looked at by the instructor but not graded. Results showed a higher class average and a lower attrition rate in the writing-integrated class than in the non-writing class. Also noted was an increase in the number of questions students asked during the class.


The author documents his observations of two high school mathematics courses and a short interview with the teacher. The author concludes that student writing is undirected, serving largely to improve communication between teacher and student and to improve classroom atmosphere.


In collaboration with an algebra teacher, the author, an English teacher, designed a three-week writing unit on solving problems with two variables. The students wrote about math concepts by expressing them exclusively with words. The experience revealed that those students
unable to explain operations were likely unable to perform them. The author concludes that writing in mathematics classes might help the large segment of students who generally neither ask for help nor fully understand instructions. Writing about operations and relations helped student comprehension. Also, this strategy can be helpful for non-mathematical students who are otherwise verbally creative.


This article draws on three years of experience in a mathematics program with student journals. The experience suggests that both mathematics students and instructors benefit from the use of journals. Journals act as a means of communication between teacher and students and help students examine their own reasoning processes. Consequently, students develop a better understanding and appreciation of both mathematics and writing.


This paper describes the usefulness of expressive journal writing in a first grade mathematics program. The students worked on activities designed to reach concepts instead of numerical symbols. The observations indicate that students made connections to previous experiences and constructed their own knowledge through writing activities. Analysis of student writing revealed four purposes of writing: self-questioning, organizing information, assimilating and accommodating information, and making guesses.

This short paper discusses journal writing in a mathematics class as a two-way conversation between the teacher and students. Students were prompted by such cues as “This is how to ...,” “The problem I am having with ...,” and “My feelings about....” Students were also given opportunities to choose their own topics. The author’s experiences indicate that the integration of writing into mathematics class helped many students improve their grades.


The author suggests that keeping learning logs or journals; writing explanations of the examples given in the class; and recording questions, confusions, interpretations, and suggestions help students learn material better and help teachers in identifying the students’ problems. The author asserts that abstract concepts become more accessible to students if more writing, reading, talking, and listening are incorporated into mathematics classroom.